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LA-UR- 01-5097 c.1

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Submitted to: 131st Annual Meeting TMS,
Indianapolis, IN
Nov. 4-8, 2001

Los Alamos
NATIONAL LABORATORY

CALCULATIONS OF THE STRESS TENSOR UNDER SYMMETRIC CYLINDRICAL SHOCK WAVE LOADING

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Abstract

The calculation of the components of the stress tensor under symmetric cylindrical shock wave loading, when the pressure impulse of cylindrical symmetry is being spread uniformly along the surface of an infinite cylindrical elastic body, have been carried out. The objective of these calculations is to assess with a sufficient approximation the stress-deformed state in samples during low intensity axis-symmetric shock wave loading. The necessity of such an assessment is grounded on a wide utilization and practical applications of shock wave axis-symmetric loading used in the explosive processing of advanced materials. The main assumptions made at the initial stage of these calculations are: elasticity and isotropy of medium, constancy of the sound speed and Lame elasticity constants, and medium boundary conditions of cylindrical symmetry. Subsequently, the removal of some assumptions during the investigation process makes possible to take into account effects engendered by boundary conditions' asymmetry and changes in the sound speed and Lame constants These changes are caused by irreversible thermal transformations going on in the medium. Well known methods for solving differential equations, such as the Fourier method, functions of Bessel, Neumann, and Hankel, equations of Helmholtz, are used in these calculations. These calculations, assuming axial symmetry, are presented as a set of simple equations where the arguments are components of the stress tensor and the solution of this set, for this specific case, gives all the components of the stress tensor.

Introduction

A number of investigations have been dedicated to the study of the disturbance processes which take place in materials as a result of the effect of a quickly changing explosive pressure[1-3]. This process covers many complex phenomena, and depending on the loading intensity level, it may be assumed that the solid body behaves either as an absolutely elastic one, or loses entirely its hardness and behaves as a liquid. Depending on this assumption, and for the analysis of the ongoing phenomena, every researcher faces the necessity to chose one of two alternative theories: the elasticity or the hydrodynamics one. The analysis of the last publications [3-6] shows that more and more authors are inclined to describe these phenomena applying the elasticity theory. Evidently this is linked with the necessity of a more precise theoretical description of the experimental results obtained during the shock loading of materials (such as a formation of new phases, texture, directed deformation of the crystalline lattice, etc.). Intensively running shear deformations dictate that we must take into account the value of and the relationships between the components of the normal and shear stresses. This issue is particularly important during the material treatment with the sliding front of the detonation wave that due to its configuration, intensity value and operational usability is quite attractive. The present article contains results obtained as an attempt to calculate the components of the stress tensor, assuming axial loading symmetry and that pressure is being spread uniformly with a constant rate at the storage capsule/sample boundary.

Results and Discussion

Let us define the stress components when, during the axis-symmetric loading, the pressure impulse initiated upon the cylinder's side surface moves along the axis with a constant speed. During these calculations, the following basic assumptions are made: (a) the ambient space is isotropic and elastic; (b) the ambient space parameters do not change during the effects of the impulse process; (c) the boundary conditions as well as the ambient space have cylindrical symmetry; and (d) axial symmetry is assumed as well as cylindrical symmetry.

It is to be mentioned that in the next stages of theses investigations, some of the restrictions mentioned above could be removed, in particular (b) and (d) ones. It will make possible to forecast effects engendered by the boundary conditions' asymmetry and to take into account changes of sound speeds and Lame elastic constants provoked by irreversible thermal transformations during the ambient space loading.

The basic equation, satisfied by the displacement vector has the following form:

$$\rho\frac{\partial^2 \vec{u}}{\partial t^2} = \frac{E}{2(1+\sigma)}\Delta\vec{u} + \frac{E}{2(1+\sigma)(1-\sigma)}\,\mathrm{grad}\,\mathrm{div}\,\vec{u}\,. \qquad (1)$$

Taking into account that

$$c_t^2 = \frac{E}{2\rho(1+\sigma)} = \frac{\eta}{\rho}; \qquad c_l^2 = \frac{E}{2(1+\sigma)(1-2\sigma)} = \frac{\lambda+2\mu}{\rho} \qquad (2)$$

where c_t and c_l stand for relatively sound's transversal and longitudinal speeds, then equation. (1) acquires the following form:

$$\ddot{\vec{u}} = c_t^2 \Delta \vec{u} + (c_l^2 - c_t^2) \operatorname{grad} \operatorname{div} \vec{u}\,. \tag{3}$$

Without any restrictions, $\vec{u}$ may be presented as a sum of two items:

$$\vec{u} = \vec{u}_l + \vec{u}_t\,, \tag{4}$$

where, $\operatorname{div} \vec{u}_t = 0 \Leftrightarrow \vec{u}_t = \operatorname{rot} \vec{A}$ $\qquad (5_t)$

and, $\operatorname{rot} \vec{u}_l = 0 \Leftrightarrow \vec{u}_l = \operatorname{grad} \varphi\,.$ $\qquad (5_l)$

Substituting (4) into (3) and taking into account conditions ($5_{l,t}$), we have:

$$\ddot{\vec{u}}_t + \ddot{\vec{u}}_l = c_t^2 \Delta(\vec{u}_l + \vec{u}_t) + (c_l^2 - c_t^2) \operatorname{grad} \operatorname{div} \vec{u}_l\,. \tag{6}$$

Let us effect by "div" operator on the both parts of the equation. With regard for condition (5_t), we will have:

$$\operatorname{div} \ddot{\vec{u}}_l = c_t^2 \Delta \operatorname{div} \vec{u}_l + (c_l^2 - c_t^2) \operatorname{div} \operatorname{grad} \operatorname{div} \vec{u}_l\,,$$

Whence

$$\operatorname{div}(\ddot{\vec{u}}_l - c_l^2 \Delta \vec{u}_l) = 0\,. \tag{7}$$

Accordingly to the condition (5_l), we have also an equation:

$$\operatorname{rot}(\ddot{\vec{u}}_l - c_l^2 \Delta \vec{u}_l) = 0\,. \tag{7'}$$

So, we have the vector

$$\xi_l = \ddot{\vec{u}} - c_l^2 \Delta \vec{u}_l\,,$$

which satisfies the conditions:

$$\operatorname{div} \vec{\xi}_l = 0\,, \ \operatorname{rot} \vec{\xi}_l = 0$$

in all points of the cylinder, including surface.

Such a vector can be only parallel to the axis of cylinder and does not depend on the distant to it (generally, time dependent), then

$$\vec{\xi}_l = \vec{k}\, f_l(t), \tag{8}$$

where $\vec{k}$ is the ort of Oz axis (cylinder's axis). So, from, (7), (7') and (8) equations we obtain:

$$\ddot{\vec{u}}_l - c_l^2 \Delta \vec{u}_l = \vec{k}\, f_l(t)\,. \tag{9}$$

Similarly, operating by rot on the both sides of equation. (6) and with regard for condition (5_l) we obtain the following equation:

$$\ddot{\vec{u}}_t - c_t^2 \Delta \vec{u}_t = \vec{k}\, f_t(t)\,. \tag{10}$$

The f_l functions presented in equations (9) and (10) are arbitrary ones so far. Thus, for transverse as well as for longitudinal waves we have equations of common type, and with the correspondent boundary conditions that they can be solved as a product:

$$\vec{u}_j(\vec{r},t) = \tilde{\vec{u}}_j(\vec{r})\psi_j(t) \quad (j = l, t) \tag{11}$$

(that is Fourier method). Then:

$$\psi''(t)\tilde{\vec{u}}(\vec{r}) - c^2\psi(t)\Delta\tilde{\vec{u}}(\vec{r}) = kf(t)\,, \tag{12}$$

Equation. (12) gives the scalar equation for it's longitudinal (z) component:

$$\psi''(t)\tilde{u}_z(\vec{r}) - c^2\psi(t)\Delta\tilde{u}_z(\vec{r}) = f(t)\,, \tag{13}$$

and gives the homogeneous equation for transverse (x and y) components:

$$\psi''\tilde{\vec{u}}_\perp(\vec{r}) - c^2\psi(t)\Delta\tilde{\vec{u}}_\perp(\vec{r}) = 0\,. \tag{14}$$

The equation. (14) reduces to two equations in r and t variables:

$$\frac{\psi''(t)}{\psi(t)} = -\omega^2; \tag{15}$$

$$\frac{\omega^2}{c^2}\tilde{\vec{u}}_\perp(\vec{r}) + \Delta\tilde{\vec{u}}_\perp(\vec{r}) = 0\,. \tag{16}$$

From the equation. (15) we have:

$$\psi(t) = \psi_+ e^{i\omega t} + \psi_- e^{-i\omega t} \tag{17}$$

Substituting (17) into (13) we obtain:

$$\omega^2\tilde{u}_z(\vec{r}) + c^2\Delta\tilde{u}_z(\vec{r}) = -\frac{f(t)}{\psi(t)}, \tag{18}$$

hence, variables' separation is made in the equation (18), that indicates that so far the undetermined function $f(t)$ has to satisfy the following conditions:

$$f(t) = f_0\,\psi(t) = f_0(\psi_+ e^{i\omega t} + \psi_- e^{i\omega t}), \tag{19}$$

while for the function $u_z(\vec{r})$ we obtain:

$$\omega^2 \tilde{u}_z(\vec{r}) + c^2 \Delta \tilde{u}_z(\vec{r}) = -f_0. \tag{20}$$

Let us designate:

$$\omega^2/c^2 = k^2 \tag{21}$$

and write down the equations (16) (Helmholtz equation) and (20) for functions $\tilde{u}_x(\vec{r})$, $\tilde{u}_y(\vec{r})$ and $\tilde{u}_z(\vec{r})$ in cylindrical coordinates:

$$k^2\tilde{u}_r(r,\alpha,z) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\tilde{u}_r\right) + \frac{1}{r^2}\frac{\partial^2\tilde{u}_r}{\partial\alpha^2} + \frac{\partial^2\tilde{u}_r}{\partial z^2} - \frac{1}{r^2}\tilde{u}_r - \frac{2}{r^2}\frac{\partial\tilde{u}_\alpha}{\partial\alpha} = 0, \tag{22_r}$$

$$k^2\tilde{u}_\alpha(r,\alpha,z) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\tilde{u}_\alpha\right) + \frac{1}{r^2}\frac{\partial^2\tilde{u}_\alpha}{\partial\alpha^2} + \frac{\partial^2\tilde{u}_\alpha}{\partial z^2} - \frac{1}{r^2}\tilde{u}_\alpha + \frac{2}{r^2}\frac{\partial^2\tilde{u}_r}{\partial\alpha} = 0, \tag{22_α}$$

$$k^2\tilde{u}_z(r,\alpha,z) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\tilde{u}_z\right) + \frac{1}{r^2}\frac{\partial^2\tilde{u}_z}{\partial\alpha^2} + \frac{\partial^2\tilde{u}_z}{\partial z^2} = -\frac{f_0}{c^2}. \tag{22_z}$$

If the axial symmetry is disturbed: $\frac{\partial u_\alpha}{\partial\alpha} \neq 0$, then, let us multiply equation (22α) on $i = \sqrt{-1}$ and add to equation (22_r).

We obtain ($W \equiv \tilde{u}_r + i\tilde{u}_\alpha$; $\tilde{u}_r = \mathrm{Re}W$; $\tilde{u}_\alpha = \mathrm{Im}W$)

$$k^2 W + \Delta W - \frac{1}{r^2}W + \frac{2i}{r^2}\frac{\partial}{\partial\alpha}W = 0. \tag{23}$$

Separating variables in the equation (23) $W(r,\alpha,z) = \tilde{W}(r,\alpha)Z(z)$, we get two independent equations:

$$Z'' + \varsigma^2 Z = 0, \tag{24_1}$$

$$(\chi^2 - \frac{1}{r^2})\tilde{W} + \Delta_\perp \tilde{W} = 0, \tag{24_2}$$

where:

$$\chi^2 = k^2 - \varsigma^2. \tag{25}$$

The solution of the equation (24_1) gives:

$$Z(z)=Z_+e^{i\varsigma z}+Z_-e^{-i\varsigma z}\,. \tag{26}$$

Separating variables in the equation (24_2) $\widetilde{W}(r,\alpha)=R(r)A(\alpha)$ we obtain:

$$A''+2iA'+h^2A=0\,; \tag{27}$$

$$R''+\frac{R'}{r}+(\chi^2-\frac{h^2+1}{r^2})R=0\,. \tag{28}$$

The solutions of Bessel equation are

$$R(r)=\begin{cases}CI_\nu(\chi r)+DN_\nu(\chi r)\\ \widetilde{C}H_\nu^{(1)}(\chi r)+\widetilde{D}H_\nu^{(2)}(\chi r)\,.\end{cases} \tag{29}$$

Here $\nu^2=h^2+1$, and $I_\nu(x)$, $N_\nu(x)$, $H_\nu^{(1)}(x)$ and $H_\nu^{(2)}(x)$ stand for Bessel, Neumann, Hankel, I kind and Hankel II kind functions, respectively. There is a connection between these solutions:

$$H_\nu^{(1,2)}(x)=I_\nu(x)\ \pm iN_\nu(x) \tag{30}$$

and they satisfy the boundary conditions:

$$I_\nu(0)=0;\qquad N_\nu(0)=-\infty\,. \tag{31}$$

In order to get finite value at $r=0$ (on the axis of cylinder) we have to assume $D=0$), hence,

$$R(r)=CI_\nu(\chi r)\,. \tag{32}$$

The general solution of equation.(27) is

$$A(\alpha)=A_1e^{i(\nu-1)\alpha}+A_2e^{-i(\nu+1)\alpha} \tag{33}$$

The $A(\alpha)$ function must be periodic $A(\alpha)=A(\alpha+2\pi)$ in order to be single valued. So we get

$$\nu=n=0;\pm 1;\pm 2... \tag{34}$$

Because of (26) and (32)-(34) the solution of equation (23) has the form:

$$W(r,\alpha,z)=R(r)A(\alpha)Z(z)=e^{-i\alpha}I_n(\chi r)(\widetilde{Z}_+e^{i\varsigma z}+\widetilde{Z}_-e^{-i\varsigma z})(\widetilde{A}_1e^{in\alpha}+\widetilde{A}_2e^{-in\alpha}) \tag{35}$$

Re and Im parts of this expression give us the functions $\widetilde{u}_r$ and $\widetilde{u}_\alpha$:

$$\widetilde{u}_r(r,\alpha,z) = \mathrm{Re}W = I_n(\chi\ r)(B_1\cos(z\varsigma\,(n-1)\alpha+\delta_1)+B_2\cos(z\varsigma+(n+1)\alpha+\delta_2)+ \\ +B_3\cos(z\varsigma\ -(n+1)\alpha+\delta_3)+B_4\cos(z\varsigma-(n-1)\alpha+\delta_4))\,, \tag{36}$$

$$\widetilde{u}_\alpha(r,\alpha,z) = \mathrm{Im}W = I_n(\chi\ r)(B_1\sin(z\varsigma+(n-1)\alpha+\delta_1)-B_2\sin(z\varsigma+(n+1)\alpha+\delta_2)+ \\ +B_3\sin(z\varsigma\ -(n+1)\alpha+\delta_3)-B_4\sin(z\varsigma\ -(n-1)\alpha+\delta_4))\,, \tag{37}$$

Let us devise variables in homogeneous equation

$$k^2\widetilde{u}_z+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\widetilde{u}_z}{\partial r})+\frac{1}{r^2}\frac{\partial^2\widetilde{u}_z}{\partial\alpha^2}+\frac{\partial^2\widetilde{u}_z}{\partial z^2}=0\,, \tag{38}$$

corresponding to the equation (22_z). For the solution $\widetilde{u}_z^0(r,\alpha,z)=R_z(r)A_z(\alpha)Z_z(z)$ of (38) after the simplification we get the equation

$$k^2r^2+\frac{r}{R_z}\frac{d}{dr}(r\frac{dR_z}{dr})+\frac{A_z''}{A_z}+r^2\frac{Z_z''}{Z_z}=0\,, \tag{39}$$

which can be split in to independent variable equations:

$$A_z''+a^2A_z=0\,, \tag{40_1}$$

$$(k^2-\varsigma_z^2-\frac{a^2}{r^2})R_z+\frac{1}{r}\frac{d}{dr}(r\frac{dR_z}{dr})=0\,, \tag{40_2}$$

$$Z_z''+\varsigma_z^2Z_z=0\,, \tag{40_3}$$

with the solutions:

$$A_z(\alpha)=A_+e^{-ia\alpha}+A_-e^{-ia\alpha}\,, \tag{41_1}$$

$$Z(z)=Z_+e^{i\varsigma_z z}+Z_-e^{-i\varsigma_z z}\,, \tag{41_2}$$

As well as above, single valuence of the function $A_z(\alpha)$, $A_z(\alpha)=A_z(\alpha+2\pi)$ lead to:

$$e^{2\pi ia}=1,\ \ a=n=0,\pm1,\pm2... \tag{42}$$

Hence equation. (40_2) reduces to the Bessel equation, which has a solution

$$R_z(r)=C_zI_n(\chi_z r)\,,$$

finite on the *Oz*-axis of the cylinder (at $r=0$). Here

$$\chi_z^2=k^2-\varsigma_z^2 \tag{43}$$

Thus, a general solution of homogeneous equation (38) is:

$$u_z^0(r,\alpha,z) = I_n(\chi_z r)(A_+ e^{-ia\alpha} + A_- e^{-ia\alpha})(Z_+ e^{i\varsigma_z z} + Z_- e^{-i\varsigma_z z}). \tag{44}$$

So, general solution of inhomogeneous equation (22_z) is:

$$\tilde{u}(r,\alpha,z) = u_z^0(r,\alpha,z) + \left(-f_0/c^2\right)\int G(r',z')\,dV', \quad (dV' = r'dr'dr'dz') \tag{45}$$

where $G(r,z)$ is the corresponding Green function which satisfies the equation:

$$(k^2 + \Delta)G(r,z) = \frac{1}{r}\delta(r)\delta(z)\delta(\alpha) \tag{46}$$

As it is known, the solution of equation (46) is

$$G(\vec{r} - \vec{r}') = -\frac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}, \tag{47}$$

where

$$|r'| = \sqrt{r^2 + z^2}, \qquad |\vec{r} - \vec{r}'| = \sqrt{r^2 + r'^2 + (z - z')^2 - 2rr'\cos(\alpha - \alpha')}.$$

So, from (45), we get

$$\tilde{u}(r,\alpha,z) = u_z^0(r,\alpha,z) + \frac{f_0}{4\pi c^2}\int_{V'} \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dV'. \tag{48}$$

Taking into account that the integration in (48) is executing on infinite volume of the cylinder and using the formulae ([7], 8.421.(11), 6.684.(1,2) and [8], 9.30.1), after simplifications we get (R stands for the radius of the cylinder):

$$I = i\pi\int_0^R r'dr'\int_0^{2\pi} d\alpha\; H_0^{(1)}\left(k\sqrt{r^2 + r'^2 + (z-z')^2 - 2rr'\cos\alpha}\right) =$$

$$= 2i\pi^2\left[\int_0^R r'dr' I_0(kr')H_0^{(1)}(kr) + \int_0^R r'dr' I_0(kr)H_0^{(1)}(kr')\right] = -\frac{4\pi}{k^2} + \frac{2i\pi^2}{k}RH_1^{(1)}(kR)I_0(kr).$$

The result is not depend on z. This means that the second term in the right side of (48) describing the oscillations of the cylinder, as of rigid body, along Oz-axis. It is clear that the amplitude of such oscillations must be zero: $f_0 = 0$, according to transition symmetry of the problem with respect to Oz axis, as well as according to the energy conservation low. Hence, taking into account (11), (17), (35), and (44) we get all three components of the displacement vector:

$$u_\alpha = I_n(\chi r)\,\mathrm{Re}\left\{e^{-i\alpha}(\tilde{Z}_+ e^{i\varsigma z} + \tilde{Z}_- e^{-i\varsigma z})(\tilde{A}_+ e^{in\alpha} + \tilde{A}_- e^{-in\alpha})(\psi_+ e^{i\omega t} + \psi_- e^{-i\omega t})\right\},$$

$$u_\alpha = I_n(\chi r)\,\mathrm{Im}\left\{e^{-i\alpha}(\tilde{Z}_+ e^{i\varsigma z} + \tilde{Z}_- e^{-i\varsigma z})(\tilde{A}_+ e^{in\alpha} + \tilde{A}_- e^{-in\alpha})(\psi_+ e^{i\omega t} + \psi_- e^{i\omega t})\right\},$$

$$u_z = I_m(\chi r)\,\mathrm{Re}\left\{(Z_+ e^{i\varsigma z} + Z_- e^{-i\varsigma z})(A_+ e^{im\alpha} + A_- e^{-im\alpha})(\psi_+ e^{i\omega t} + \psi_- e^{-i\omega t})\right\} \qquad (49)$$

$(n, m = 0, \pm 1, \pm 2, \ldots)$

The components of stress tensor can be founded from (49). At first, let us use the expressions for displacement tensor (see i.e. [9]) in cylindrical coordinates:

$$u_{rr} = \frac{\partial u_r}{\partial r}, \quad u_{\alpha\alpha} = \frac{1}{r}\frac{\partial u_\alpha}{\partial \alpha} + \frac{u_r}{r}, \quad 2u_{r\alpha} = \frac{\partial u_\alpha}{\partial r} - \frac{u_\alpha}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \alpha},$$

$$u_{zz} = \frac{\partial u_z}{\partial z}, \quad 2u_{\alpha z} = \frac{1}{r}\frac{\partial u_z}{\partial \alpha} + \frac{\partial u_\alpha}{\partial z}, \quad 2u_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r},$$

so, taking into account (35), one can get:

$$u_{rr} - u_{\alpha\alpha} + 2iu_{r\alpha} = \left(\frac{\partial}{\partial r} + \frac{i}{\partial r}\frac{\partial}{\partial \alpha} - \frac{1}{r}\right)(u_r + iu_\alpha) = \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial r} - \frac{1}{r}\right)W, \qquad (50)$$

$$2u_{rz} + 2iu_{\alpha z} = \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \alpha}\right)u_z + \frac{\partial}{\partial z}(u_r + iu_\alpha) = \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \alpha}\right)u_z + \frac{\partial}{\partial z}W, \qquad (51)$$

$$u_{zz} = \frac{\partial}{\partial z}u_z \qquad (52)$$

It is easy to check, that the stress tensor's components are

$$\sigma_{rr} - \sigma_{\alpha\alpha} + 2i\sigma_{r\alpha} = \frac{E}{1+\sigma}[u_{rr} - u_{\alpha\alpha} + 2iu_{r\alpha}] = \frac{E}{1+\sigma}\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \alpha} - \frac{1}{r}\right)W, \qquad (53)$$

$$2\sigma_{rz} + 2i\sigma_{\alpha z} = \frac{E}{1+\sigma}\left[\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \alpha}\right)u_z + \frac{\partial}{\partial z}W\right], \qquad (54)$$

$$\sigma_{zz} + i(\sigma_{rr} + \sigma_{\alpha\alpha}) + \frac{i\sigma - 1}{1 - 2\sigma}2\sigma_{\alpha r} =$$

$$= \frac{E}{1+\sigma}\frac{1}{1-2\sigma}\{(1 - \sigma + 2i\sigma)u_{zz} + (\sigma + i)(u_{rr} + u_{\alpha\alpha}) + (i\sigma - 1)2u_{\alpha r}\} =$$

$$= \frac{E}{(1+\sigma)(1-2\sigma)}\left\{(1 - \sigma + 2i\sigma)\frac{\partial u_z}{\partial z} + (\sigma + i)\left(\frac{\partial}{\partial r}W + \frac{i}{r}\frac{\partial}{\partial \alpha}\overline{W} + \frac{1}{r}\overline{W}\right)\right\}, \qquad (55)$$

where $\overline{W}$ stands for complex conjugate of W.

Real and imagine parts of the left-hand-sides of these three complex formulae unique defined all six (real) components of stress tensor. Thus, (53)-(55) formulae give us expressions for the six real functions, or the- six real components of stress tensor - using only two (complex) functions u_z and W and their derivatives.

In the case of axial symmetry, these expressions simplifies:

$$\sigma_{rr}-\sigma_{\alpha\alpha}+2i\sigma_{r\alpha}=\frac{E}{1+\sigma}\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)W,$$

$$2\sigma_{rz}+2i\sigma_{\alpha z}=\frac{E}{1+\sigma}\left(\frac{\partial}{\partial r}u_z+\frac{\partial}{\partial z}W\right),$$

$$\sigma_{zz}+i(\sigma_{rr}+\sigma_{\alpha\alpha}+\frac{i\sigma-1}{1-2\sigma}2\sigma_{\alpha r}=\frac{E}{(1+\sigma)(1-2\sigma)}\left\{(1-\sigma+2i\sigma)\frac{\partial u_z}{\partial z}+(\sigma+i)\left(\frac{\partial W}{\partial r}+\frac{\overline{W}}{r}\right)\right\}. \quad (56)$$

Solving the system of equations (56), one found an explicit form for these expressions:

$$\sigma_{rr}=\frac{E}{(1+\sigma)(1-2\sigma)}\left[\frac{1+\sigma}{2}\mathrm{Im}\frac{\partial u_z}{\partial z}-\sigma\,\mathrm{Re}\frac{\partial u_z}{\partial z}+(1-\sigma)\,\mathrm{Re}\frac{\partial W}{\partial r}+\sigma\,\mathrm{Re}\frac{W}{r}\right],$$

$$\sigma_{\alpha\alpha}=\frac{E}{(1+\sigma)(1-2\sigma)}\left[\frac{1+\sigma}{2}\mathrm{Im}\frac{\partial u_z}{\partial z}-\sigma\,\mathrm{Re}\frac{\partial u_z}{\partial z}+\sigma\,\mathrm{Re}\frac{\partial W}{\partial r}+(1-\sigma)\,\mathrm{Re}\frac{W}{r}\right],$$

$$\sigma_{zz}=\frac{1}{2}\frac{E}{(1+\sigma)}\mathrm{Re}\left(\frac{\partial u_z}{\partial z}+\frac{\partial W}{\partial z}\right),$$

$$\sigma_{r\alpha}=\frac{E}{2(1+\sigma)}\mathrm{Im}\left[\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)W\right],$$

$$\sigma_{\alpha z}=\frac{1}{2}\frac{E}{(1+\sigma)}\mathrm{Im}\left(\frac{\partial u_z}{\partial r}+\frac{\partial W}{\partial z}\right),$$

$$\sigma_{rz}=\frac{1}{2}\frac{E}{(1+\sigma)}\mathrm{Re}\left(\frac{\partial u_z}{\partial r}+\frac{\partial W}{\partial z}\right). \quad (57)$$

The formulae (57) strongly decrease the volume of necessary numerical calculations (and computing time, correspondingly) and, due to decreasing intermediate operations, improve an accuracy of results. Using formulae (17), (35) and the third equation of system (49) give us possibility to calculate stress tensor's components without of calculation the displacement tensor explicitly.

Conclusions

The results obtained in this research make it possible to calculate the components of the stress tensor without the explicit calculation of the displacement tensor. This methodology strongly decreases the volume of necessary numerical calculations (and thus computing time) and, due to the decreasing intermediate operations, improves an accuracy of results.

The analyses performed show that, without loosing generality, it is possible to calculate all the components of the tensor of stress-deformed axial-symmetric sample, in any case of dynamical loading.

This method allows us to generalize the conditions of investigation and, at the next stage, take into account the non-symmetric boundary conditions and the effects of the non-reversal thermal processes, which take place in real experiments.

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The analyses performed show that, without loosing generality, it is possible to calculate all the components of the tensor of stress-deformed axial-symmetric sample, in any case of dynamical loading.

This method allows us to generalize the conditions of investigation and, at the next stage, take into account the non-symmetric boundary conditions and the effects of the non-reversal thermal processes, which take place in real experiments.

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